Spherical Models for *The Gaia Challenge*

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ABSTRACT

We provide discrete (6-D) samplings of 32 unique phase-space distribution functions (DFs) calculated for various potentials (assumed to be dominated by a spherical dark matter halo), tracer density profiles, and tracer velocity anisotropies assumed to be of Osipkov-Merrit form: $\beta(r) \equiv 1 - \bar{v^2}_\theta / \bar{v^2}_r = r^2/(r^2 + r_a^2)$. Additionally we provide 3600 mock data sets that optionally include any and/or all (at the user's discretion) of the following phenomena: observational errors, foreground contamination, spectral-index (i.e., a proxy for metallicity), binary orbital motions, perspective-induced velocity gradients due to systemic motion transverse to the line of sight, and chemo-dynamically independent stellar sub-populations. These mock data span a range of realistic sample sizes and include various levels of intrinsic overlap among up to two stellar sub-populations. Data can be downloaded from the GaiaChallenge wiki.

Subject headings: Gaia Challenge, Spherical Models

1. Discrete Samplings of Tracer Distribution Functions

We follow Walker & Peñarrubia (2011) in considering dynamical tracer populations (i.e., stellar populations) that are distributed according to a generalized Hernquist density profile (Hernquist 1990; Zhao 1996),

$$\nu_*(r) = \nu_0 \left(\frac{r}{r_*}\right)^{-\gamma_*} \left[1 + \left(\frac{r}{r_*}\right)^{\alpha_*}\right]^{(\gamma_* - \beta_*)/\alpha_*}, \qquad (1)$$

and dark matter halos with density profiles that take the same form,

$$\rho_{\rm DM}(r) = \rho_0 \left(\frac{r}{r_{\rm DM}}\right)^{-\gamma_{\rm DM}} \left[1 + \left(\frac{r}{r_{\rm DM}}\right)^{\alpha_{\rm DM}}\right]^{(\gamma_{\rm DM} - \beta_{\rm DM})/\alpha_{\rm DM}}. \qquad (2)$$

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These profiles have independent parameters specifying normalization, scale radius, inner logarithmic slope (γ, subscripts omitted for brevity), outer logarithmic slope (β), and the sharpness (α) of the transition between the two slopes.

We consider dynamical models in which the central slope of the dark matter density profile takes values of either $\gamma_{\rm DM} = 0$ or $\gamma_{\rm DM} = 1$. We hold fixed other halo parameters at scale radius $r_{\rm DM} = 1$ kpc, outer slope $\beta_{\rm DM} = 3$ and $\alpha_{\rm DM} = 1$. For the tracer populations we consider 'generalized' Plummer profiles that have structural parameters $\alpha_* = 2$, outer slope $\beta_* = 5$, inner slopes $\gamma_* = 0.1, 1.0$ and a range of scale radii $r_*/r_{\rm DM} = 0.1, 0.25, 0.5, 1$ corresponding to various degrees of 'embeddedness' within the dark matter halo.

We consider the family of spherical, anisotropic distribution functions discussed by Osipkov (1979) and Merritt (1985). These models have velocity distributions with anisotropy profiles of the form $\beta_{\rm ani}(r) \equiv 1 - \bar{v_\theta^2}/\bar{v_r^2} = r^2/(r^2 + r_a^2)$. We consider values for the anisotropy radius r_a that give the stellar subcomponent a velocity distribution that either is isotropic at all radii ($r_a = \infty$) or gradually changes from isotropic at small radii to radially anisotropic at large radii ($r_a = r_*$). Having specified the profiles $\nu(r)$, $\rho(r)$ and $\beta_{\rm ani}(r)$ for each stellar subcomponent in each dark matter halo, we use Equation 11 of Merritt (1985) to calculate the corresponding phase-space distribution functions. We check this calculation by performing N-body simulations in which stars orbit within the adopted potential and have initial positions/velocities drawn from the calculated distribution function. These simulations show no significant departures from the initial dynamical configuration after 100 crossing times, indicating that the calculated distribution functions indeed correspond to equilibrium dynamical models.

Table 1 lists the grid of input parameters that specifies 32 unique dynamical models that we use to represent individual dSph stellar subcomponents.

Files containing discrete samplings corresponding to a single tracer population are available at the GaiaChallenge wiki (see **spherical_df.tar.gz**) and have names

gsAAA_bsBBB_rcrsCCC_rarcDDD_EEEE_FFFFmpc3_df.dat,

where capital letters encode intrinsic properties of the model:

AAA= $100\gamma_*$
BBB= $10\beta_*$
CCC= $100 r_*/r_{DM}$
DDD= $100 r_a/r_*$
EEEE="core" for $\gamma_{DM} = 0$, "cusp" for $\gamma_{DM} = 1$
FFFF= $1000\rho_0$.

Notice the multiplicative coefficients, which are present in order to eliminate periods from the filenames.

In these files, each row of six columns contains an independent random sampling of the 6-D phase-space distribution. Columns give the following information:

1. x position with respect to center (units of kpc)
2. y position with respect to center (kpc)
3. z position with respect to center (kpc)
4. v_x velocity with respect to system mean (km/s)
5. v_y velocity with respect to system mean (km/s)
6. v_z velocity with respect to system mean (km/s)

2. Mock Data Sets

Using the discrete samplings described above, we generate mock data sets that include realistic phenomena such as observational errors, foreground contamination, spectral-index (i.e., a proxy for metallicity), binary orbital motions, perspective-induced velocity gradients due to systemic motion transverse to the line of sight, and chemo-dynamically independent stellar subpopulations. For completeness we allow all mock data sets to have contributions from two stellar sub-components as well as a foreground component. In order to grant the user maximum flexibility, we identify the population from which each star is drawn.

Given the grid of models specified by Table 1, there are 320 unique ways to combine two tracer populations that share the same potential. We perform ten realizations of each combination, giving a total of 3200 mock data sets. In setting up a given realization we draw stellar population parameters randomly from uniform distributions within the following limits:

- sample sizes $3 \leq \log_{10}[N_1+N_2+N_{\rm MW}] \leq 4$ (similar to the available samples)
- member fractions $0.4 \leq (N_1+N_2)/(N_1+N_2+N_{\rm MW}) \leq 0.9$
- subcomponent fractions $0.1 \leq N_1/(N_1+N_2) \leq 0.9$
- mean systemic velocities (heliocentric rest frame) $0 \leq \langle V \rangle/(\mathrm{kms}^{-1}) \leq 250$
- mean spectral index $0.3 \leq \langle W' \rangle_1/\text{Å} \leq 0.5$ for the 'metal-rich' subcomponent
- mean spectral index separation $0 \leq (\langle W' \rangle_1 - \langle W' \rangle_2)/\text{Å} \leq 0.25$
- proper motions $-100 \leq \mu_\alpha/(\mathrm{mas/cent}) \leq +100$ and $-100 \leq \mu_\delta/(\mathrm{mas/cent}) \leq +100$.
- binary fractions $0 \leq f_b \leq 1$, where f_b is the fraction of sampled stars to which we add binary motion

We place half (randomly selected) of the synthetic ‘dSphs’ at the (3D) position of the Fornax dwarf spheroidal (dSph; α = 02 : 39 : 59, δ = –34 : 27 : 00, D = 138 kpc) and the other half at the location of Sculptor dSph (α = 01 : 00 : 09, δ = –33 : 42 : 30, D = 79 kpc; Mateo 1998).

With the above stellar population parameters specified for a given realization, we then use an accept/reject algorithm to draw the appropriate numbers of positions and velocities from discrete random samplings of the appropriate 6D distribution function. We then project the positions and velocities along the line of sight in order to mimic observables. Next we assign reduced Mg indices, W' (Walker, Mateo & Olszewski 2009), to each star according to whether it is drawn from a relatively ‘metal-rich’ or ‘metal-poor’ sub-population. We assign W' values to the metal-rich and metal-poor member stars by drawing values from Gaussian distributions with variances $\sigma^2_{W',1} = \sigma^2_{W',2} = 0.02$ Å^2 and means drawn randomly from the ranges specified above. To the line-of-sight velocities of all member stars we apply redshifts $\langle V \rangle_{\alpha_*,\delta_*}$ appropriate to the systemic 3D space motion and line of sight (Walker et al. 2008) and add velocities corresponding to binary orbital motions (see below). Finally, we scatter all velocities and W' values according to actual measurement errors drawn randomly from the dSph data of (Walker, Mateo & Olszewski 2009, median errors are $\epsilon_V \sim 2$ km s^{-1} and $\epsilon_{W'} \sim 0.01$Å).

To stars drawn from a ‘foreground’ contamination component we assign positions drawn randomly from a uniform spatial distribution (within the projected position of the outermost member star) and assign velocities drawn randomly from the Besançon model of Milky Way stars (filtered by photometric criteria for selecting dSph red giants) along the line of sight to the either (chosen randomly in each realization) the Fornax or the Sculptor dSph. To foreground stars we assign W' values and associated errors drawn directly from measurements of probable ($P_{\rm mem} < 0.1$) foreground stars in the data of Walker, Mateo & Olszewski (2009).

2.1. Binary Orbital Motions

As mentioned above, to the line-of-sight velocity of a fraction f_b of sampled points we add binary motion given by

$$v_b = \frac{2\pi a_1 \sin i}{P\sqrt{1-e^2}}\left[\cos(\theta+\omega)+e\cos\omega\right], \tag{3}$$

where a_1, P and e are the semimajor axis, period and eccentricity, respectively, of the primary’s orbit, i is the inclination of the orbital plane with respect to the line of sight, θ is the phase with respect to periastron, and ω is the longitude of periastron.

Following McConnachie & Côté (2010), we assume the primary has mass $m1 = 0.8M_\odot$ and adopt log-normal distributions for the mass ratio $q \equiv m_2/m_1$ and for the period distribution, as fit by Duquennoy & Mayor (1991) to binaries in the field. That is,

$$\frac{dN}{dq} \propto \exp\left[-\frac{1}{2}\frac{(q-\overline{q})^2}{\sigma_q^2}\right], \tag{4}$$

and

$$\frac{dN}{d\log_{10}P} \propto \exp\left[-\frac{1}{2}\frac{\left(\log_{10}P-\overline{\log_{10}P}\right)^2}{\sigma^2_{\log_{10}P}}\right] \qquad (5)$$

with $\overline{q} = 0.23$, $\sigma_q = 0.42$, $\overline{\log_{10}[P]} = 4.8$ and $\overline{\sigma_{\log_{10}P}} = 2.3$, where P is measured in days (Duquennoy & Mayor 1991). Following McConnachie & Côté (2010), we set $q_{min} = 0.1$ so that the secondary always has mass larger than the threshold for hydrogen burning.

Again following McConnachie & Côté (2010), we consider two possible distributions for eccentricity. First, we consider circular orbits ($e = 0$ throughout), and second we consider a 'thermal' distribution (Heggie 1975):

$$\frac{dN}{de} \propto 2e. \qquad (6)$$

We draw inclinations with probability proportional to $sin(i)$. We draw the orientation, ω, with uniform probability between 0 and π. We draw the phase with probability that is proportional to the inverse of the angular velocity, $\dot{\theta}^{-1}(m,P,e)$. We add no binary motion to the two velocity components (v_x, v_y) transverse to the line of sight, as the long time baseline required to measure proper motions will effectively average over the binary phase in a way that instantaenous redshifts do not.

2.2. Mock Data Files

Files containing mock data sets are available at the GaiaChallenge wiki (see **core_mock.tar.gz** and **cusp_mock.tar.gz**) and have names **[model_name]_6d.mem2**, where

[model_name]=c1_AAA_BBB_CCC_DDD_EEE_c2_FFF_GGG_HHH_III_JJJ_NNN,

and
AAA= $100\gamma_*$, for member component 1
BBB= $10\beta_*$ for member component 1
CCC= $r_*/10$, for member component 1, units of pc
DDD= $100r_a$, for member component 1 (a value of "inf" implies isotropy)
EEE="core" for $\gamma_{\rm DM} = 0$, "cusp" for $\gamma_{\rm DM} = 1$
FFF= $100\gamma_*$, for member component 2
GGG= $10\beta_*$, for member component 2
HHH= $r_*/10$, for member component 2, units of pc
III= $100r_a$, for member component 2 (a value of "inf" implies isotropy)
JJJ= "core" for $\gamma_{\rm DM} = 0$, "cusp" for $\gamma_{\rm DM} = 1$.

Each line in these files gives information about a given star sampled from the appropriate distribution function. The (x,y,z) coordinate system has origin at the center of the galaxy and the $+z$ coordinate increases with distance along the line of sight. Columns give:

1. x position (pc)
2. y position (pc)
3. z position (pc)
4. v_x velocity (km/s; includes observational error)
5. v_y velocity (km/s; includes observational error)
6. v_z velocity (km/s; includes observational error, perspective effect due to systemic proper motion and los-component of binary-orbital motion)
7. $\delta(v_x)$ (km/s; observational error for x velocity in column 4)
8. $\delta(v_y)$ (km/s; observational error for y velocity in column 5)
9. $\delta(v_z)$ (km/s; observational error for z velocity in column 6)
10. v_x velocity as drawn directly from DF (i.e., before including observational errors and perspective effect; km/s)
11. v_y velocity as drawn directly from DF (i.e., before including observational errors and perspective effect; km/s)
12. v_z velocity as drawn directly from DF (i.e., before including observational errors, perspective effect and/or binary motions; km/s)
13. v_b (km/s; LOS velocity due to binary orbital motion; this is already included in the v_z velocity given in column 6)
14. Mg index (Angstroms)
15. $\delta(Mg)$ (Angstroms, observational error for Mg index in column 15)
16. Right Ascension (radians)
17. Declination (radians)
18. Right Ascension of center of galaxy (radians; will be either Fornax's or Sculptor's)
19. Declination of center of galaxy (radians; will be either Fornax's or Sculptor's)
20. probability of membership (evaluated using EM algorithm of Walker et al. (2009)
21. Which component is star drawn from (1=member component 1, 2=member component 2, 3=foreground)?

The associated file [model_name]_6d.samplepars gives parameters that fully specify the input model. Columns of its single line give:

1. systemic v_z velocity (km/s)
2. $\langle Mg \rangle_1 = \langle Mg \rangle_2$ (Angstroms; difference in mean Mg index between inner (subscript 1) and outer (subscript 2) components)
3. N_{sample}, number of stars in sample
4. $N_{\mathrm{members}}/N_{\mathrm{sample}}$; stars that are not members are drawn from foreground model
5. fraction of members that belong to member component 1 (as many as two member components are considered)
6. μ_α (mas/century); systemic proper motion in RA direction
7. μ_δ (mas/century); systemic proper motion in Dec direction
8. γ_*, component 1

9. β_*, component 1
10. r_*, component 1 (pc)
11. r_a/r_*, component 1 (assumes Osipkov-Merritt anisotropy profile; note that isotropic models have a large value of 10^4)
12. γ_*, component 2
13. β_*, component 2
14. r_*, component 2 (pc)
15. r_a/r_*, component 2 (assumes Osipkov-Merritt anisotropy profile, note that isotropic models have a large value of 10^4)
16. $\gamma_{\rm DM}$
17. $\beta_{\rm DM}$
18. $r_{\rm DM}$ (pc)
19. $\alpha_{\rm DM}$
20. ρ_0 ($M_\odot$/pc^3)
21. distance to center of galaxy (pc)
22. not applicable
23. not applicable
24. $\langle Mg \rangle_1$ (Angstroms)
25. binary fraction–i.e., fraction of member stars for which z velocity received contribution from binary motions.

Notice that the **[model_name]_6d.mem2** files provide sufficient information that the user can remove either of the member components as well as foreground, and remove observational errors, perspective effects and binary motions as well.

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Table 1. Tests on synthetic data: grid of input parameters for dynamical test models

Profile	Parameter	values considered
Stellar Subcomponent (Eq. 1)		
	$r_*/r_{\rm DM}$	$0.10, 0.25, 0.50, 1.0$
	α_*	2
	β_*	5
	γ_*	$0.1, 1.0$
	r_a/r_*	$1, \infty$
Dark Matter Halo (Eq. 2)		
	$\rho_0/(M_\odot {\rm pc}^{-3})$	0.064 for $\gamma_{DM} = 1$, 0.40 for $\gamma_{DM} = 0$
	$r_{\rm DM}/{\rm kpc}$	1
	$\alpha_{\rm DM}$	1
	$\beta_{\rm DM}$	3
	$\gamma_{\rm DM}$	$0, 1$